Comment on 'A close examination of the motion of an adiabatic piston' by Eric A. Gislason [Am. J. Phys. 78, 995–1001 (2010)]

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(Dated: December 23, 2010)

Abstract

A recent paper by Gislason published in AJP deals with the celebrated example of the so-called "adiabatic piston", a system involving two ideal gases contained in a horizontal cylinder and separated by an insulating piston that moves without friction.[1] While the analysis presented in that paper is rather comprehensive, very interesting and useful as a teaching tool, it can be somewhat misleading if not taken within its appropriate context. As a matter of fact, the evolution to equilibrium involves two phases, a faster one leading to the equalization of pressures, and a slower one bringing the system to identical temperatures. Although Gislason addresses only the first process, we note that the final state after the second phase, the evolution to equal temperatures once the pressures are the same, *is described by thermodynamics*. Therefore, the discussion of the adiabatic piston given by Gislason can and should be enriched, in order to promote a proper and general view of thermodynamics.

I.   INTRODUCTION

In a recent paper published in AJP, Eric A. Gislason puts forward a detailed analysis of the motion of the "adiabatic piston" problem, consisting of two subsystems of the same ideal gas contained in a horizontal cylinder with insulating walls.[1] Gislason makes several important points and elaborates on the *first process*, which brings the piston to rest when the pressures of the two gases will be equal. Significant physical insight given by Gislason is the damping of the piston motion as a result of the *dynamic pressure* on the piston, "because the pressure is greater when the piston is moving towards the gas than when the piston is moving away from the gas."[1] Gislason cites several authors who have pointed out that "temperature and pressure fluctuations in the two gases will slowly act to bring the two temperatures to equality."[1] He correctly states that the "time scale for this slow process is much longer than the time scale for the piston to come to rest"[1] and warns the reader that this slow process is not discussed in the paper. Gislason asserts that "thermodynamics cannot predict what the final temperatures will be",[1] which is right only within the framework of the analysis of the first process. Moreover, he adds that "to achieve complete equilibrium the piston must be able to conduct energy, which cannot occur for an adiabatic piston".[1] The latter assertion cannot be read in the context of the study of the faster process and, as detailed in the next section, is not rigorously valid if one keeps in mind the second process as well. Notice that we find it very interesting to present the analysis of the first process as done by Gislason, simply students should be aware of the approximations involved and the conceptual problems it hides in some measure. The purpose of this comment is to contribute to clarify this issue, by using the formalism of thermodynamics to extend the investigation to the second process as well.

An affirmation on the impossibility of thermodynamics to predict the final equality of temperatures is a critical step, since a kinetic-statistical interpretation indeed foresees the motion of the piston until the two temperatures are equal. In this line, an intuitive and beautiful discussion of this second process is made by Feynman,[2] whereas a quantitative molecular dynamic simulation, establishing the phenomenon beyond doubt, was published by Kestemont and co-workers.[3] The message we want to convey here is that an accurate and careful use of thermodynamics must give the same final results as any kinetic simulation, as the latter is a microscopic interpretation of the results of the former. In passing, let us

announce we find that no occasion is too much to pay a tribute to the genius of Ludwig Boltzmann, and this is a perfect occasion to do so.

The remaining of this comment is structured as follows. The way in which thermodynamics may handle the "adiabatic piston" problem is shown in the next section. A short discussion and an identification of the origin of some common misunderstandings is given in section III. Finally, section IV summarises our main conclusions.

II.   THERMODYNAMICS' ANSWER TO THE PROBLEM

Let us start by giving a straight answer to the "adiabatic piston" problem, namely, what are the final pressures and temperatures of both gases, leaving further discussion for the next section. The equality of pressures is a necessary condition, usually referred to as the condition for *mechanical equilibrium*, corresponding to the first process. However, it is not sufficient for the complete equilibrium, the *thermodynamical equilibrium*, correlated to the second, slower, phase.

It is worth noting that we cannot impose $dS = 0$ once the pressures are equal,[4] although this is sometimes confused with the "adiabatic" condition (*cf.* next section). Indeed, there are configurations in the vicinity of the mechanical equilibrium, with greater global entropy, and the system will move towards these configurations. Take note that the two subsystems are connected through the conditions of constant total volume and total energy. The collisions between the gas particles and the piston will make the piston jiggle, *allowing an exchange of energy* between both gases. This energy exchanges will take place *even if the piston is not a thermal conductor*, as they are simply a result of the momentum transfer in the collisions (see the discussion by Feynman[2]). As a consequence, the system will indeed access the different available microscopic configurations and move as a result of a blind entropic process, in accordance with Boltzmann's basic ideas and his microscopic interpretation of entropy.[5,6] This is why the assertion that "to achieve complete equilibrium, the piston must be able to conduct energy, which cannot occur for an adiabatic piston" does not hold.

Taking into account the preliminary discussion above, the system is described by the

following set of equations[4]

$$dU_1 = -p_1 \, dV_1 + T_1 \, dS_1 \tag{1}$$

$$dU_2 = -p_2 \, dV_2 + T_2 \, dS_2 \tag{2}$$

Moreover, we have the condition

$$dS = dS_1 + dS_2 \geq 0 \; , \tag{3}$$

where the equality holds for the final equilibrium. Equations (1) and (2) can be written in the form

$$dS_1 = \frac{dU_1}{T_1} + \frac{p_1}{T_1} dV_1 \tag{4}$$

$$dS_2 = \frac{dU_2}{T_2} + \frac{p_2}{T_2} dV_2 \tag{5}$$

Now, the piston jiggles, but, as long as the system reaches mechanical equilibrium,

$$dE_k = -dU_1 - dU_2 = 0 \; , \tag{6}$$

where E_k is the kinetic energy of the piston. Furthermore,

$$dV = dV_1 + dV_2 = 0 \; . \tag{7}$$

Hence, $dU_2 = -dU_1$ and $dV_2 = -dV_1$. Substituting (4) and (5) in the equilibrium condition (3), we finally get

$$dS = \left(\frac{1}{T_1} - \frac{1}{T_2} \right) dU_1 + \left(\frac{p_1}{T_1} - \frac{p_2}{T_2} \right) dV_1 \equiv 0 \; . \tag{8}$$

Therefore, the solution to our problem is

$$p_1 = p_2 \tag{9}$$

$$T_1 = T_2 \tag{10}$$

and *both the mechanical and the thermodynamical equilibria are obtained.* Thermodynamics can and does predict the final variables.

III.   DISCUSSION

In the previous section we have shown that thermodynamics predicts correctly the evolution of the system to a state of equal pressures *and* equal temperatures. The reason

the contrary inaccurate statement is repeated by many authors is related to a problem of language and a misconceived notion associated with the word "adiabatic". Actually, if the piston is "adiabatic", an additional condition is often imposed, based on an erroneous physical intuition, specifically,

$$dU_i = -p_i dV_i \quad , \quad i = 1, 2 . \tag{11}$$

The argument is that, since the piston is "adiabatic", $dQ = 0$. If this would be the case we would have, substituting (11) in (8),

$$dS = -\left(\frac{1}{T_1} - \frac{1}{T_2}\right) p_1 \, dV_1 + \left(\frac{p_1}{T_1} - \frac{p_2}{T_2}\right) dV_1 \equiv 0 . \tag{12}$$

This expression would then be valid if the mechanical equilibrium $p_1 = p_2$ holds, without the need for equality of the temperatures. In fact, using $p_2 = p_1$ in (12),

$$dS = -\left(\frac{1}{T_1} - \frac{1}{T_2}\right) p_1 \, dV_1 + \left(\frac{1}{T_1} - \frac{1}{T_2}\right) p_1 \, dV_1 ,$$

which is identically zero, regardless of the values of T_1 and T_2.

The critical point here is to realise that the additional condition (11) *is extraneous to the formalism.* When the designation "adiabatic piston" is used, it is meant a piston *with zero heat conductivity.* If the piston is held in place (for instance, if it is fixed to the box by screws), then there is no heat transfer from one subsystem to the other. Even though, if the piston is released, both systems are coupled, therefore interacting and exchanging energy, as explained in the previous section, and there is "heat exchange". Trying to use the common language, we could say that *a piston which is "adiabatic" when it is fixed, no longer is "adiabatic" when it can move freely*! The condition $dQ = 0$ cannot be imposed, as it comes from a faulty instinct still somewhat related with the idea of caloric (associating "heat" to a fluid, or at least being mislead by the designations "heat" and "thermal insulator" or "adiabatic"), and completely misses the subtlety of the concept of heat.

It is not too difficult to show that equation (11) is not general and cannot be demonstrated.[4] The conservation of energy is expressed by the first part of equation (6), $dE_k + dU_1 + dU_2 = 0$. On the other hand, the work done on the piston is

$$dW = dE_k = (p_1' - p_2') \, dV_1 , \tag{13}$$

where p_1' and p_2' are *dynamic pressures* (notice that they are denoted by P_1 and P_2 in Gislason's paper[1]), *i.e.*, the pressures the gases exert on the moving piston. Therefore,

$$dU_1 + dU_2 = -(p_1' - p_2') \, dV_1 . \tag{14}$$

This does *not* imply that (11) is valid, as it does not require with generality $dU_i = -p'_i dV_i$, although this can be a good approximation during the fast process. Hence, even after the first phase, when pressures are equal but the temperatures are still different, we must write

$$dU_i = -p_i \, dV_i + T_i \, dS_i \neq -p_i \, dV_i , \tag{15}$$

and (11) is not general.

After the attainment of mechanical equilibrium the piston has no kinetic energy and the evolution to the final equilibrium verifies $dU_1 = -dU_2$, *i.e.*

$$- p_1 \, dV_1 + T_1 dS_1 = +p_2 \, dV_2 - T_2 dS_2 . \tag{16}$$

Since $p_1 = p_2$ and $dV_1 = -dV_2$, it comes

$$T_1 dS_1 = -T_2 dS_2 . \tag{17}$$

Finally, if $T_1 > T_2$, and taking into account (3), $dS_2 > 0$ and $dS_1 < 0$, although the global change of entropy is positive.[4] Accordingly, the temperatures T_2 and T_1 will slowly raise and decrease, respectively, until both temperatures become equal and full equilibrium is achieved.

IV.   CONCLUSION

A very nice paper published recently in AJP raises several interesting points on thermodynamics using the example of the "adiabatic piston".[1] As asserted in that paper, the results it obtains must be used exclusively as a description of the first process of evolution to equilibrium, leading to mechanical equilibrium. However, the slow evolution to thermodynamical equilibrium is also well described within classical thermodynamics and the complete equilibrium is in truth achieved, even if the piston is not a thermal conductor. We believe this example can be extremely useful in classroom to illustrate the subtleties around the concept of "heat", which goes beyond the first ideas leading to its introduction in Physics. Besides, it helps advancing a more general and proper view of thermodynamics, providing as well a strong link to the microscopic interpretation of entropy. Additional appreciation of the problem, including the analysis of the first phase and the damped oscillations of the piston, can be found in a paper by Mansour and co-workers[7] and in some former work of R. de Abreu.[8,9] Further discussion on the concepts of work and heat is also available from Gislason and Craig.[10]

Acknowledgments

The authors are indebted to Professor Eric A. Gislason for the very rewarding discussion and his suggestions ensuing the submission of this comment, which contributed to increase the quality and clarity of the manuscript.

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